

Famous Inequalities:

Triangular: $|x + y| \leq |x| + |y|$ or equivalently $||x| - |y|| \leq |x - y|$

In $\mathbb{R}^n$ with standard norm, it has a form:

$$\sqrt{(x_1 - y_1)^2 + \dots + (x_n - y_n)^2} \leq \sqrt{x_1^2 + \dots + x_n^2} + \sqrt{y_1^2 + \dots + y_n^2}$$

Minkowski: $\left(\sum_{i=1}^n (a_i + b_i)^p \right)^{1/p} \leq \left(\sum_{i=1}^n a_i^p \right)^{1/p} + \left(\sum_{i=1}^n b_i^p \right)^{1/p}$ for $a_i, b_i \geq 0, p > 0$

Notice that for $n = p = 2$ we have **Triangular Inequality:**

$$\sqrt{(a_1 + b_1)^2 + (a_2 + b_2)^2} \leq \sqrt{a_1^2 + a_2^2} + \sqrt{b_1^2 + b_2^2}$$

Cauchy-Schwarz: $a_1 b_1 + \dots + a_n b_n \leq \sqrt{a_1^2 + \dots + a_n^2} \sqrt{b_1^2 + \dots + b_n^2}$

(with equality iff the sequences (a_i) and (b_i) are proportional)

Bernoulli: $(1 + x)^n \geq 1 + nx$ for every integer $n \geq 0$ and real $x \geq -1$

with strict inequality $(1 + x)^n > 1 + nx$ for $n \geq 2$ and $x \neq 0$

Jensen: $\frac{\sum_{i=1}^n f(x_i)}{n} \leq f\left(\frac{\sum_{i=1}^n x_i}{n}\right)$ if f is **concave down**

$$\frac{\sum_{i=1}^n f(x_i)}{n} \geq f\left(\frac{\sum_{i=1}^n x_i}{n}\right) \text{ if } f \text{ is } \mathbf{concave up}$$

with equality iff $x_1 = x_2 = \dots = x_n$.

Example: $\frac{\ln a_1 + \dots + \ln a_n}{n} \leq \ln\left(\frac{a_1 + \dots + a_n}{n}\right)$ as $\ln x$ is concave down